Problem 47

For vectors $\vec{\mathbf{B}} = -\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$ and $\vec{\mathbf{A}} = -3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$, calculate (a) $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ and its magnitude and direction angle, and (b) $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ and its magnitude and direction angle.

Solution

Part (a)

In order to add the vectors, add their respective components.

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (-3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$$

$$= (-3 - 1)\hat{\mathbf{i}} + (-2 - 4)\hat{\mathbf{j}}$$

$$= -4\hat{\mathbf{i}} - 6\hat{\mathbf{j}}$$

Calculate the magnitude.

$$\left| \vec{\mathbf{A}} + \vec{\mathbf{B}} \right| = \sqrt{(-4)^2 + (-6)^2}$$

$$= 2\sqrt{13}$$

$$\approx 7.21$$

The direction angle of $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ is

$$\theta = \tan^{-1}\left(\frac{-6}{-4} \right) + 180° \approx 236°,$$

where 180° is added because the vector sum is in the third quadrant.

Part (b)

In order to subtract the vectors, subtract their respective components.

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = (-3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) - (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$$

$$= (-3 + 1)\hat{\mathbf{i}} + (-2 + 4)\hat{\mathbf{j}}$$

$$= -2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$$

Calculate the magnitude.

$$\left| \vec{\mathbf{A}} - \vec{\mathbf{B}} \right| = \sqrt{(-2)^2 + 2^2}$$

$$= 2\sqrt{2}$$

$$\approx 2.83$$

The direction angle of $\overrightarrow{\mathbf{A}} - \overrightarrow{\mathbf{B}}$ is

$$\theta = \tan^{-1}\left(\frac{2}{-2}\right) + 180° = 135°,$$

where 180° is added because the vector difference is in the second quadrant.